Robust Appointment Scheduling

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Outline

- **The problem**: Appointment scheduling in health care services
- **The model**: A novel robust optimization framework
- **The solution**: Global balancing heuristic, closed form optimal solution and worst case scenarios

Appointment Scheduling in Health Care Services

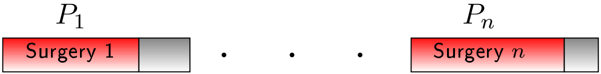
- High-cost facilities: MRI, CAT Scan, Operation rooms etc.

- Conflicting costs: Utilization of resources versus quality of service
- Uncertain processing durations

The Problem

Example: Scheduling outpatient surgeries

Given:



Example: Scheduling outpatient surgeries

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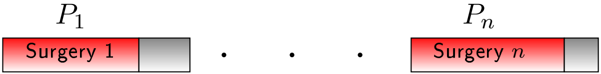
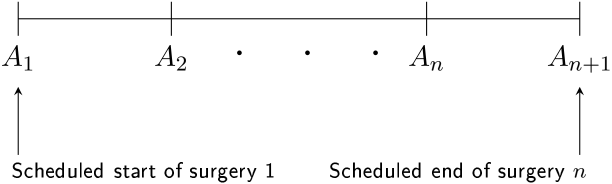


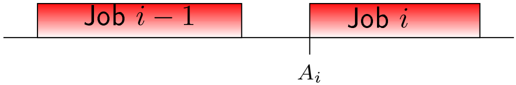
Find:



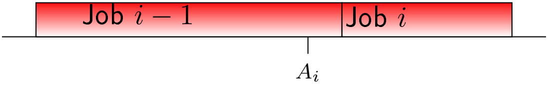
The Problem

Job processing:

- If job $i-1$ finishes before A_i, job i starts at A_i.



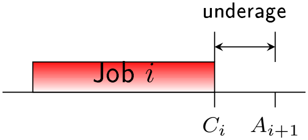
- Otherwise: job i starts immediately after completion of job $i-1$.



Costs

C_i = completion time of job i.

- $C_i < A_{i+1}$: underage cost $u_i(A_{i+1} - C_i)$. (Job i is *underaged*)



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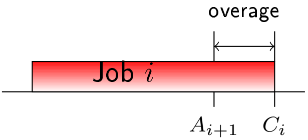

Costs

P : a given realization of processing times of jobs.

Cost function

$$F(A, P) = \sum_{i=1}^{n} \max(o_i(C_i - A_{i+1}), u_i(A_{i+1} - C_i))$$

Example 1: 3 jobs, $u = 10$, $o = 1$

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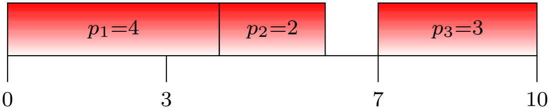

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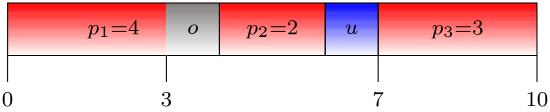

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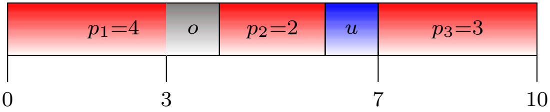
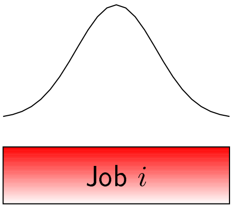


Total Cost $= 1 + 10 + 0 = 11$

Other Applications

- Project scheduling (Bendavid and Golany, 2009)
- Serial production systems (Elhafsi 2002)
- Servicing ships at seaports (Sabria and Daganzo 1989)
- Professors scheduling meeting with grad students
- ...

Existing Models



P_i: random variable

Cost function

$$F(A) = \mathbb{E}_P[F(A, P)]$$

Optimization problem: Minimize expected cost

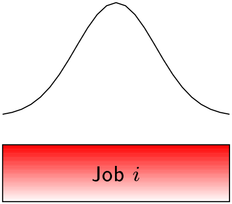
Drawback 1: Intractability

- Computing cost of a schedule may be intractable

Known methods:

- Sequential bounding algorithm (Denton and Gupta 2003)
- Monte Carlo techniques (Robinson and Chen 2003)
- Local search (Kandoorp and Koole 2007)
- Submodular function minimization (Begen and Queyranne 2009)

Drawback 2: Need for Data



P_i: random variable

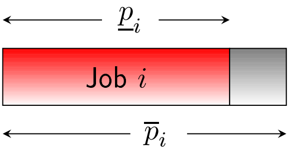
Distribution of P_i may not be known

Our Contributions

1. A robust optimization framework
2. Closed form optimal solution
3. Structural insights into the problem

The Robust Model

Given: minimum and maximum possible execution time of each job.



$\mathcal{P}$: Set of all possible realization of processing times of jobs

Robust Model

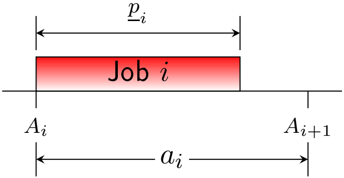
$$F(A) = \max_{P \in \mathcal{P}} F(A, P)$$

Optimization problem: minimize worst-case scenario(s) cost.

The Global Balancing Heuristic

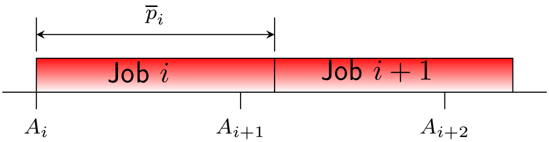
Main idea: Balance between maximum underage cost of job i, and maximum overage cost *due to* job i.

Maximum possible underage cost of job $i = u_i(a_i - \underline{p}_i)$.



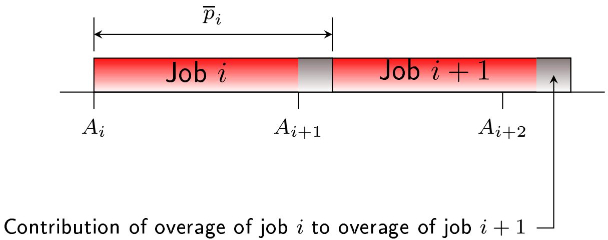
The Global Balancing Heuristic

Maximum possible contribution of job i to overage costs of all jobs succeeding i:



The Global Balancing Heuristic

Maximum possible contribution of job i to overage costs of all jobs succeeding i: $(\sum_{j=i}^{n} o_j)(\overline{p}_i - a_i)$.



The Global Balancing Heuristic

Equating maximum possible underage and overage costs:

$$u_i(a_i - \underline{p}_i) = (\sum_{j=i}^{n} o_j)(\overline{p}_i - a_i)$$

We get:

$$a_i^G = \frac{u_i\underline{p}_i + (\sum_{j=i}^{n} o_j)\overline{p}_i}{u_i + \sum_{j=i}^{n} o_j}$$

The Main Theorem

Theorem

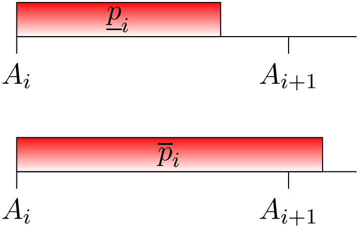
The global balancing schedule is optimal for the robust version when the underage costs of the jobs are equal.

Closed form optimal solution for robust model

Intuitive Interpretation

If job i alone is scheduled:

$$a_i^* = \frac{u_i \underline{p}_i + o_i \overline{p}_i}{u_i + o_i}$$

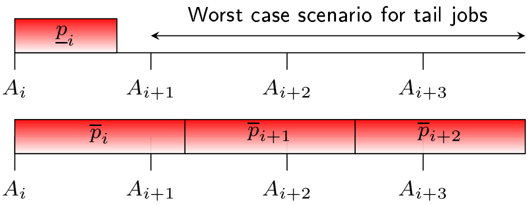


Intuitive Interpretation

However, if jobs $i+1, \ldots, n$, are to be scheduled after job i:

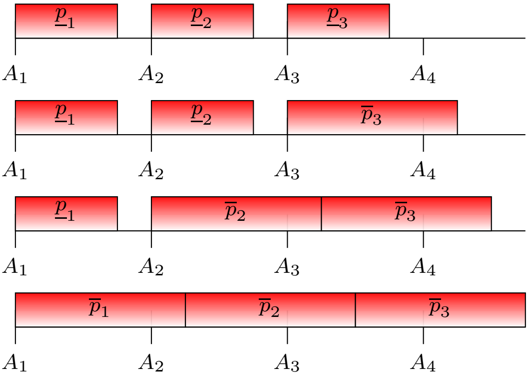
$$a_i^G = \frac{u_i \underline{p}_i + o_{\geq i} \overline{p}_i}{u_i + o_{\geq i}}$$

where $o_{\geq i} = \sum_{j=i}^{n} o_j$.



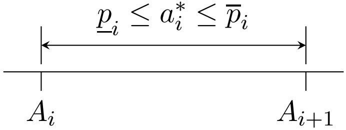
Worst Case Scenarios for the Optimal Schedule

Sequence of min-length jobs followed by max-length jobs.



Structure of the Optimal Solution

Assigned duration to job i is in interval $[\underline{p}_i, \overline{p}_i]$.

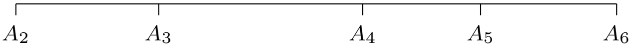


- $\underline{p}_i \leq a_i^*$: True for any instance (Begen and Queyranne 2009).
- $a_i^* \leq \overline{p}_i$: Holds only for non-decreasing u_i's.

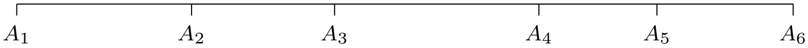
Structure of the Optimal Solution

Optimal assigned duration to a job independent of jobs preceding it

Example: Optimal solution for jobs 2-5:



If job 1 is added to front of schedule:



Comparison with Stochastic Model

Cost parameters: $u = 10$, $o = 1$

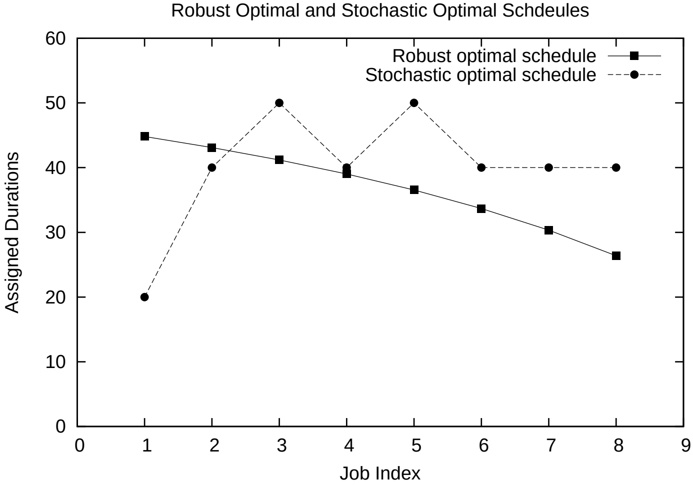
Stochastic model:

- Job durations: Discrete version Weibull distribution with $\mu = 48$ and $\sigma = 26$
- Stochastic optimal solution found using local search

Robust model:

- $\underline{p} = \mu - \sigma = 22$, $\overline{p} = \mu + \sigma = 74$

Comparison with Stochastic Model



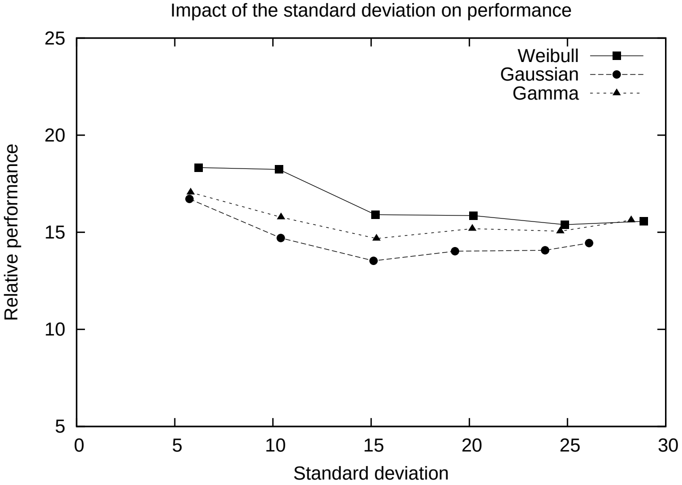
Relative Performance of Robust Model

$$\text{Relative performance} = \frac{ROB - OPT}{OPT} \times 100,$$

where

- OPT = Average cost of stochastic optimal schedule
- ROB = Average cost of robust optimal schedule

Impact of Standard Deviation



Conclusion

Summary

- Closed form solution for robust model
- Model can be used even if no historical data available
- Structural insights into the problem

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- Closed form solution for robust model
- Model can be used even if no historical data available
- Structural insights into the problem

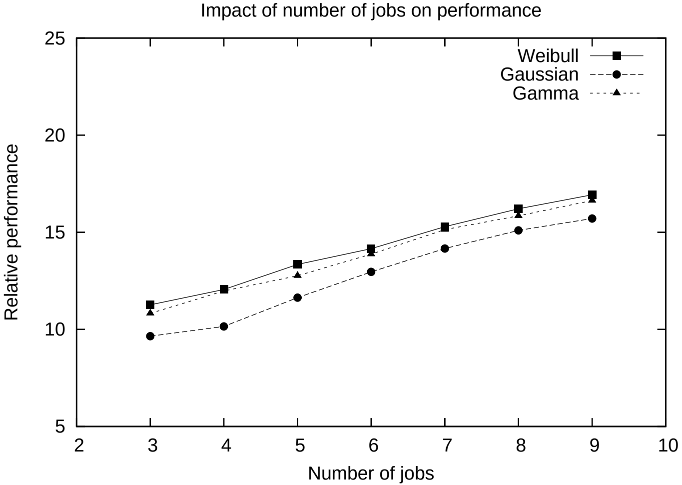
Future directions

- Solving the ordering problem
- Scheduling multiple facilities
- Incorporating no-shows and emergency jobs

Thank You!

Questions??

Impact of Number of Jobs



Impact of Underage Cost